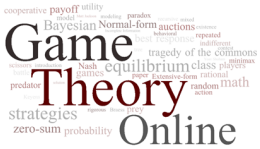
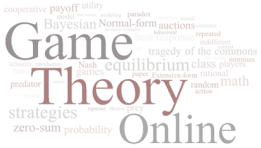


Discounted Repeated Games

Game Theory Course:
Jackson, Leyton-Brown & Shoham

Discounted Repeated Games



- The future is uncertain, we are often motivated by what happens today
- Tradeoffs of today and the future are important in how I will behave today
- Will people punish me if I misbehave today?
  - Is it in their interest?
  - Do I care?

Discounted Repeated Games

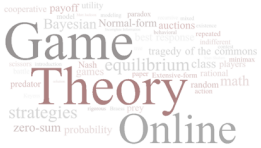
- Stage game: (N, A, u)
- Discount factors: $\beta_1, \ldots, \beta_n$, $\beta_i \in [0, 1]$
- Assume a common discount factor for now: $\beta_i = \beta$ for all i
- Payoff from a play of actions $a^1, \ldots, a^t, \ldots$:

$$\sum_t \beta_i^t u_i(a^t)$$

Histories

- Histories of length t: $H^t = \{h^t : h^t = (a^1, \ldots, a^t) \in A^t\}$
- All finite histories: $H = \cup_t H^t$
- A strategy: $s_i : H \to \Delta(A_i)$

Prisoners Dilemma

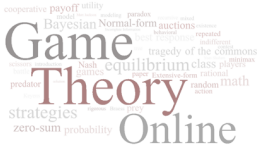


- $A_i = \{C, D\}$
- A history for three periods: $(C, C), (C, D), (D, D)$
- A strategy for period 4 would specify what a player would do after seeing $(C, C), (C, D), (D, D)$ played in the first three periods...

Subgame Perfection

- Profile of strategies that are Nash in every subgame
- So, a Nash equilibrium following every possible history
- Repeatedly playing a Nash equilibrium of the stage game is always a subgame perfect equilibrium of the repeated game (Stop and check this!)

Repeated Prisoner's Dilemma



- Cooperate as long as everyone has in the past
- Both players defect forever after if anyone ever deviates: Grim Trigger

	C	D
C	3,3	0,5
D	5,0	1,1

Repeated Prisoner's Dilemma

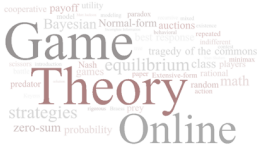
- Let's check that nobody wants to deviate if everyone has cooperated in the past:
- Cooperate: $3 + \beta 3 + \beta^2 3 + \beta^3 3... = \frac{3}{1-\beta}$
- Defect: $5 + \beta 1 + \beta^2 1 + \beta^3 1... = 5 + \beta \frac{1}{1-\beta}$

	C	D
C	3,3	0,5
D	5,0	1,1

Repeated Prisoner's Dilemma

- Let's check that nobody wants to deviate if everyone has cooperated in the past:
- Cooperate: $3 + \beta 3 + \beta^2 3 + \beta^3 3... = \frac{3}{1-\beta}$
- Defect: $5 + \beta 1 + \beta^2 1 + \beta^3 1... = 5 + \beta \frac{1}{1-\beta}$
- Difference: $-2 + \beta 2 + \beta^2 2 + \beta^3 2... = \beta \frac{2}{1-\beta} - 2$
- Difference is nonnegative if $\beta \frac{2}{1-\beta} - 2 \geq 0$ or $\beta \geq (1 - \beta)$, so $\beta \geq 1/2$
- Need to care about tomorrow at least half as much as today!

Repeated Prisoner's Dilemma



- What if we make defection more attractive:

	C	D
C	3,3	0,10
D	10,0	1,1

Repeated Prisoner's Dilemma

- Let's check that nobody wants to deviate if everyone has cooperated in the past:
- Cooperate: $3 + \beta 3 + \beta^2 3 + \beta^3 3... = \frac{3}{1-\beta}$
- Defect: $10 + \beta 1 + \beta^2 1 + \beta^3 1... = 10 + \beta \frac{1}{1-\beta}$
- Difference: $-7 + \beta 2 + \beta^2 2 + \beta^3 2... = \beta \frac{2}{1-\beta} - 7$
- Difference is nonnegative if $\beta \frac{2}{1-\beta} - 7 \geq 0$ or $2\beta \geq 7(1-\beta)$, so $\beta \geq 7/9$
- Need to care about tomorrow at least 7/9 as much as today!

Discounted Repeated Games

- Basic logic:
  - Play something with relatively high payoffs, and if anyone deviates
  - Punish by resorting to something that
    - has lower payoffs (at least for that player)
    - and is credible: it is an equilibrium in the subgame.